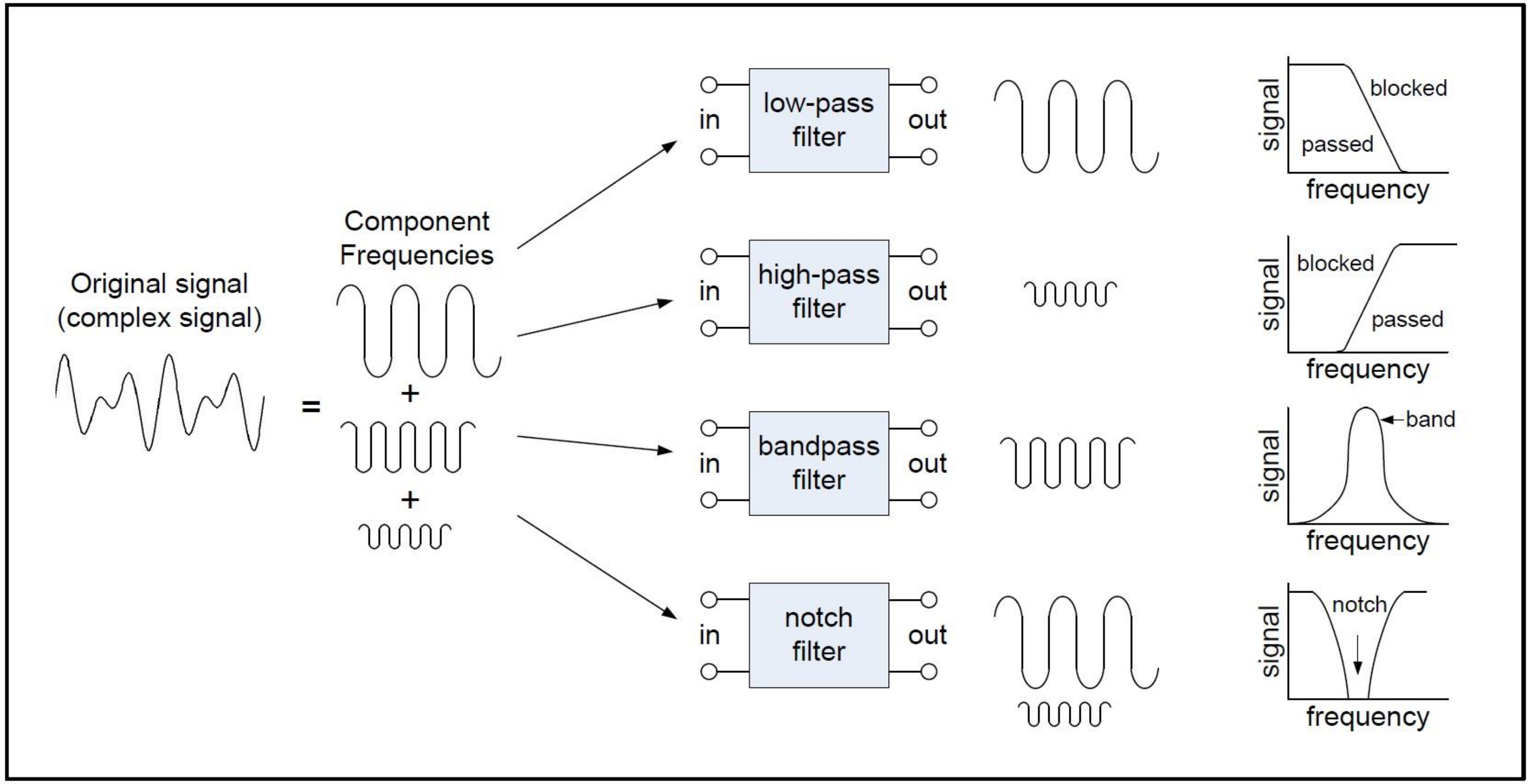
Introduction to electronic filters – some theory and practical experiments

Introduction

A filter is a circuit capable of passing (or amplifying) certain frequencies while attenuating other frequencies. Thus, a filter can extract important frequencies from signals that also contain undesirable or irrelevant frequencies.



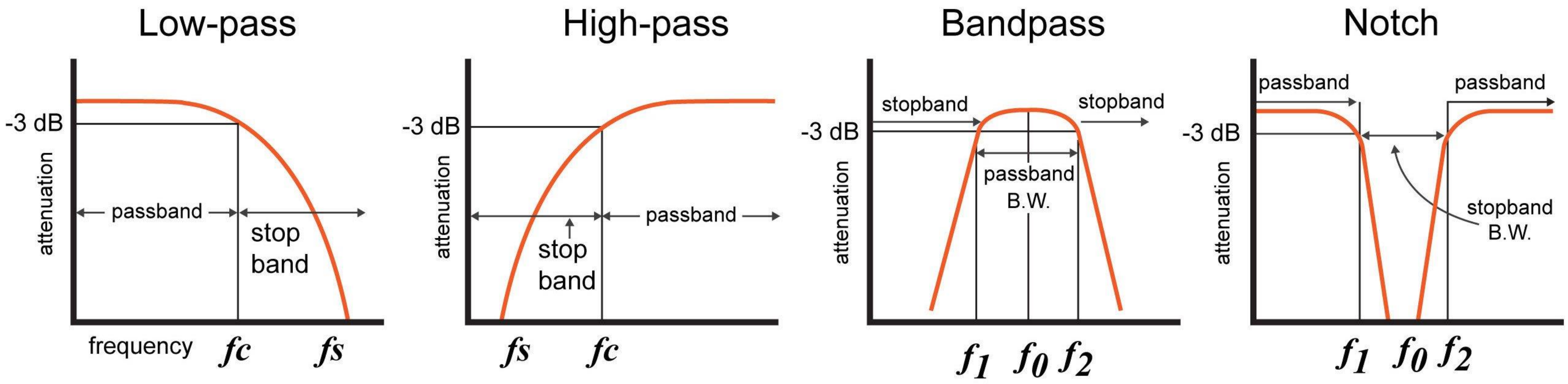
Key terms

Response curve (V_{out} / V_{in}) with attenuation in dB

f_c, f_1, f_2 = corner frequency (in Hz or ω); f_{3dB} = minus 3 dB frequency = cutoff frequency = half-power frequency = $Vout = \frac{Vin}{\sqrt{2}}$

f_s = stop band frequency (defined attenuation has been reached)

B.W. = bandwidth or β



RC Lowpass

Simplest, 1st order low pass filter.

$$V_{out} = Vin * \frac{Z_2}{Z_1 + Z_2}$$

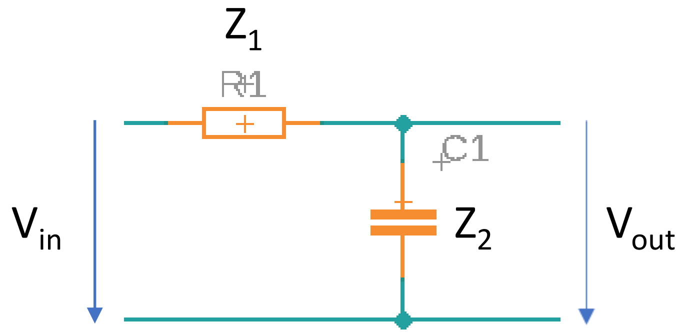
$$Z_2 \rightarrow C_1 \rightarrow X_c = \frac{1}{2\pi f C_1}$$

$$V_{out} = Vin * \frac{Z_2}{Z}$$

$$Z = \sqrt{R_1^2 + X_c^2}$$

Vin = 10V
R1 = 4.7kΩ
C1 = 47nF
fc = $\frac{1}{2\pi R_1 C_1}$ = 720Hz

f = 100Hz: V_{out} = 9.9V
f = 10kHz: V_{out} = 0.72V



Voltage transfer function: $H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$ --> $H_{RC}(\omega) = \frac{1}{1 + j\omega RC}$

Cut-off frequency def. = $|H_{RC(\omega)}| = \frac{1}{\sqrt{2}} = \mathbf{0.707}$ --> $j\omega RC = 1$ --> $\boldsymbol{fc} = \frac{\mathbf{1}}{\mathbf{2\pi RC}}$

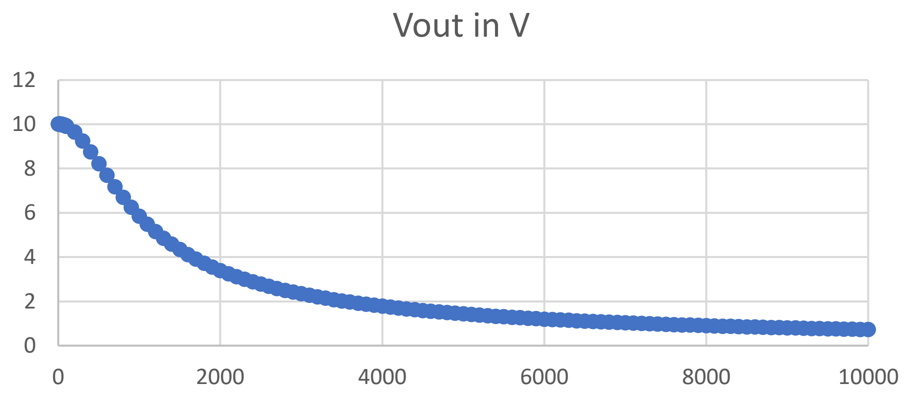
Phase at cut-off: $H_{RC}(\omega) = 1/(1+j\frac{1}{RC}RC)$

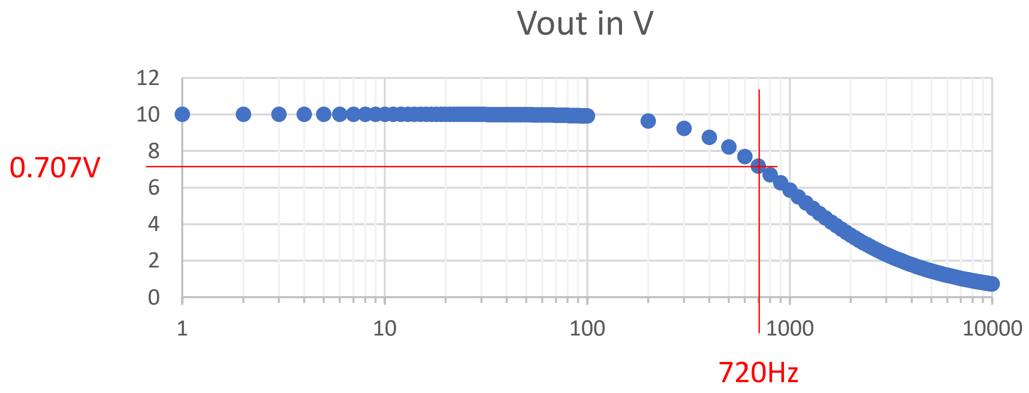
```
octave:1> 1/(1+j)
ans =  0.5000 - 0.5000i
```

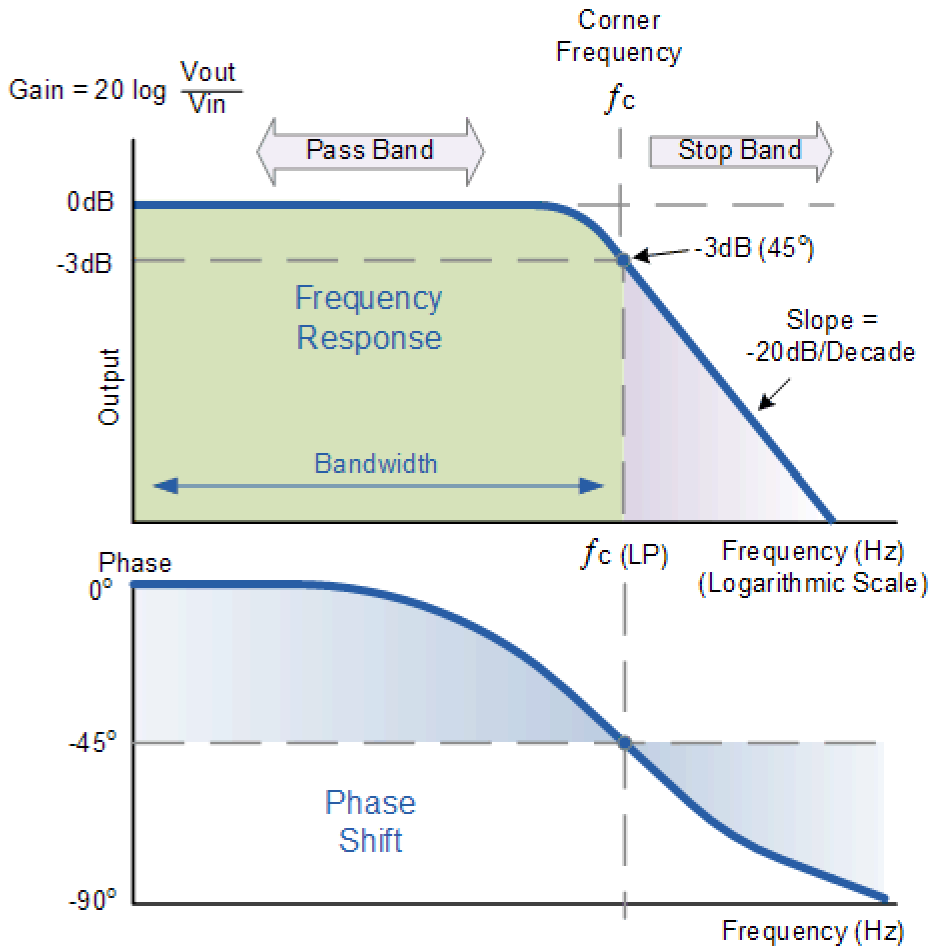
--> cos(1) = 45°

1st order RC lowpass

-20db/decade = -6dB/octave
Octave: 1 octave is the ratio of 2 of a frequency
Decade: 1 decade is the ratio of 10 of frequency
An octave is 3.332 times in a decade ($2^x = 10$)

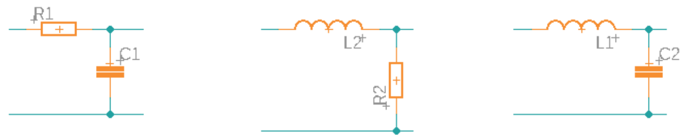






To the bench…

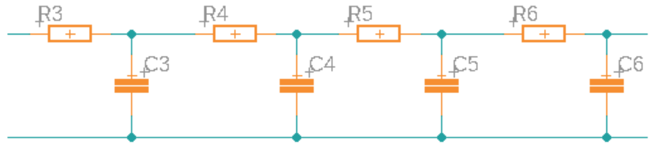
Lowpass “higher orders”



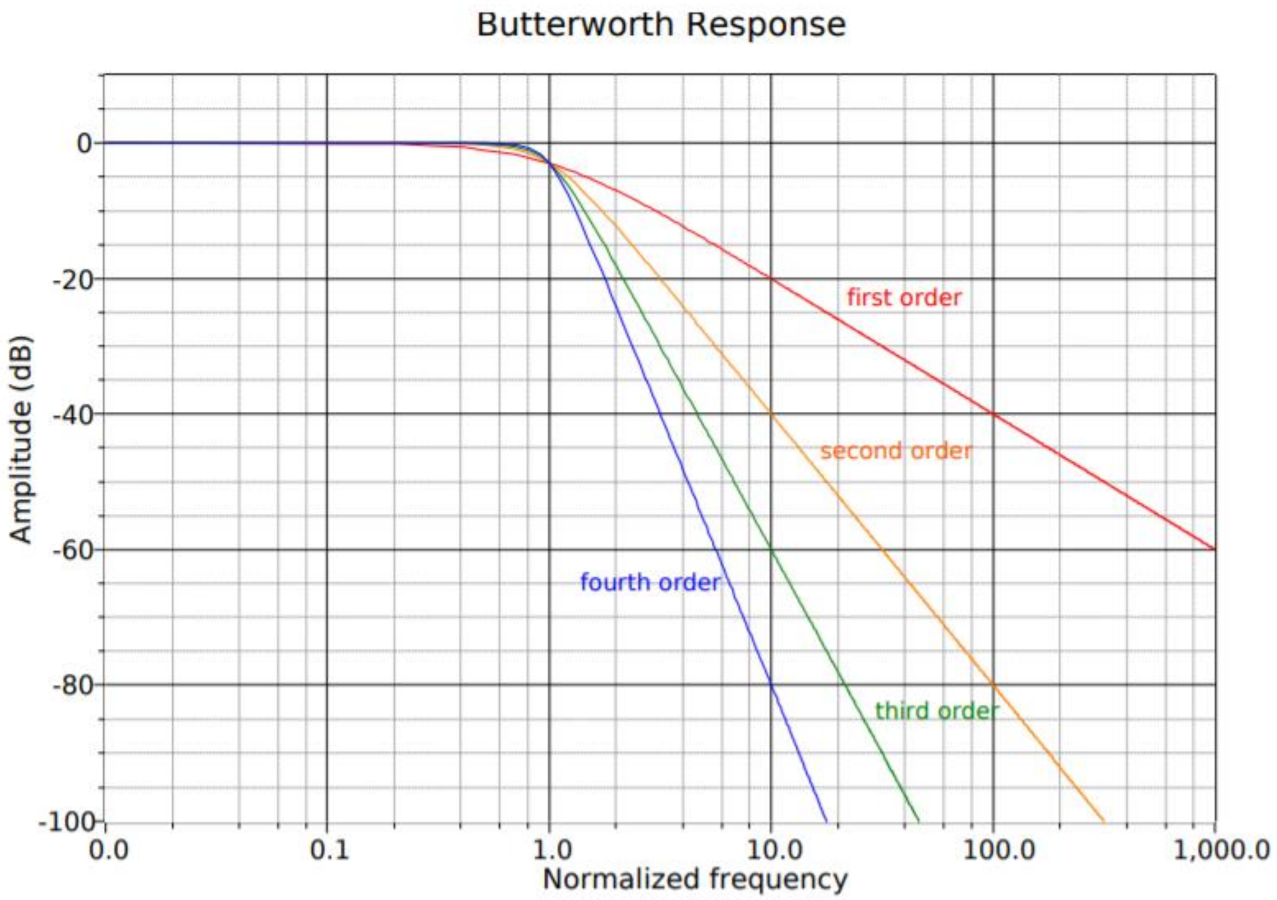
$$fc = \frac{1}{2\pi RC} \qquad fc = \frac{R}{2\pi L} \qquad fc = \frac{1}{2\pi\sqrt{LC}}$$

1st order 2nd order

X



4th order



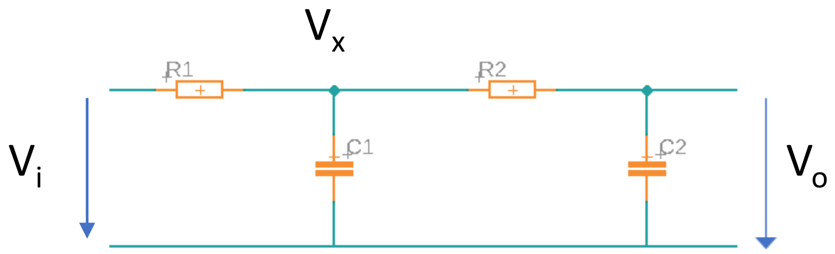
Higher the order (or poles) of a filter -> faster rolloff rate

Filter order -> derived from transfer function

Filter order -> minimum number of reactive components requir

https://eng.libretexts.org/Bookshelves/Electrical_Engineering/Electronics/Operational_Amplifiers_and_Linear_Integrated_Circuits_-_Theory_and_Application_(Fiore)/11%3A_Active_Filters/11.04%3A_Section_4-

2nd order RC lowpass



$$f_n = \frac{1}{(2\pi * \sqrt{R_1 R_2 C_1 C_2})}$$

= natural frequency (resonance) not -3dB cutoff frequency!

I: KCL in V_o:

$$\frac{V_o - V_x}{R_2} + sC_2 V_o = 0$$

$$V_x = V_o(1 + sR_2C_2)$$

II: KCL in V_x:

$$\frac{V_x - V_i}{R_1} + \frac{V_x - V_o}{R_2} + sC_1 V_x = 0$$

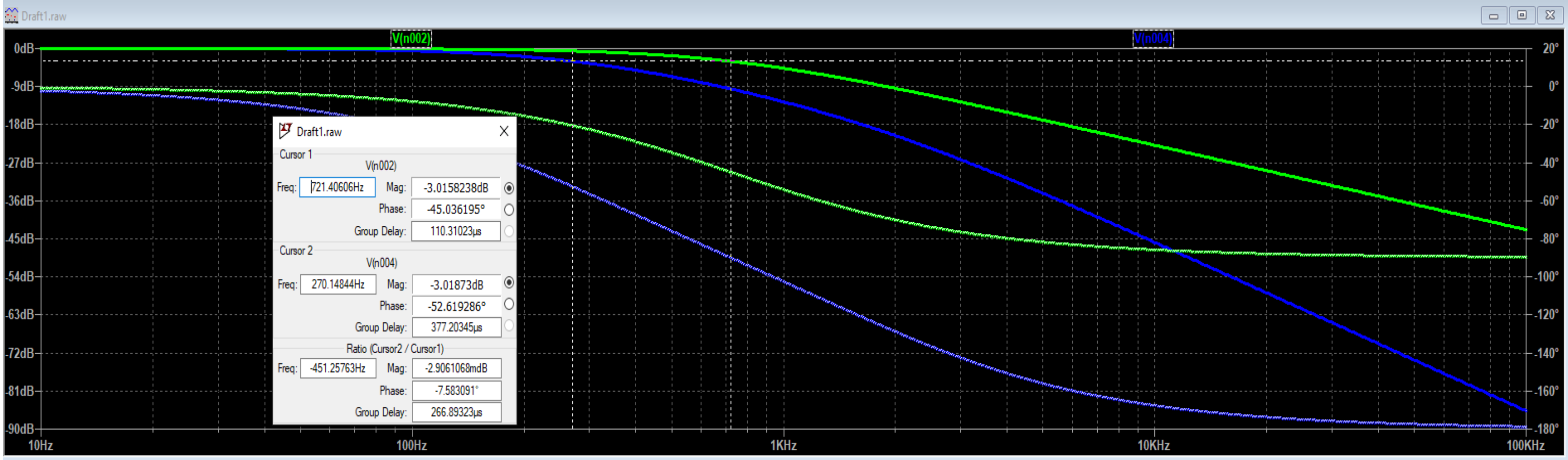
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 R_1 R_2 C_1 C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

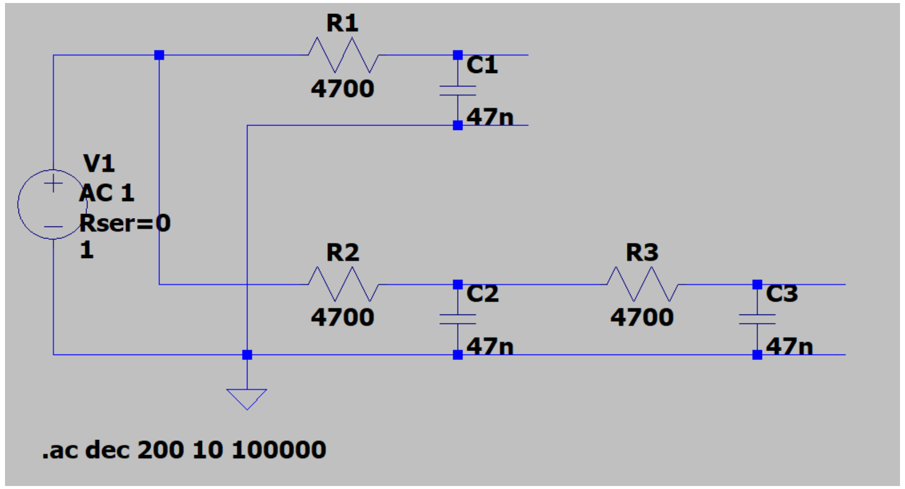
Take aways:

- Loading of second stage changes reduces cut-off freq.
- Loading of the output affects the filter behavior as well!

https://electronics.stackexchange.com/questions/152159/deriving-2nd-order-passive-low-pass-filter-cutoff-frequency

1st vs. 2nd order lowpass filter simulated



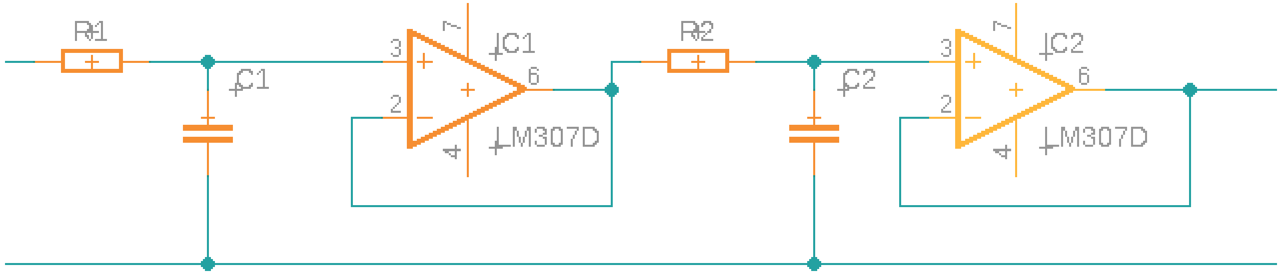


$$f_n = \frac{1}{(2\pi * \sqrt{R_1 R_2 C_1 C_2})}$$

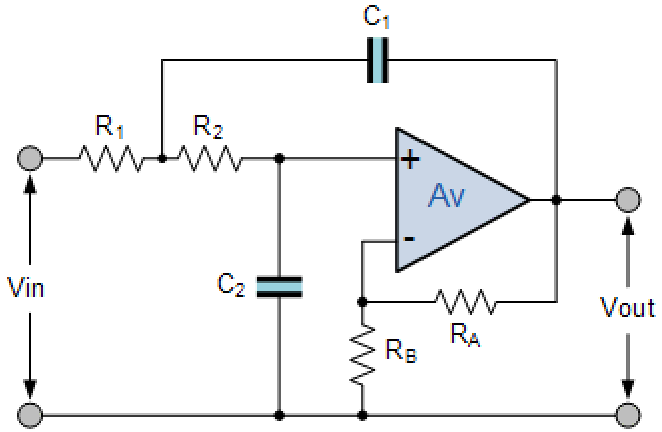
fn = 720Hz

fc = 270Hz!!

RC lowpass 2nd order; decoupling the outputs



Second Order Low Pass Filter



Sallen-Key Topology

$$\text{Gain (Av)} = 1 + \frac{R_A}{R_B}$$

If Resistor and Capacitor values are different:

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

If Resistor and Capacitor values are the same:

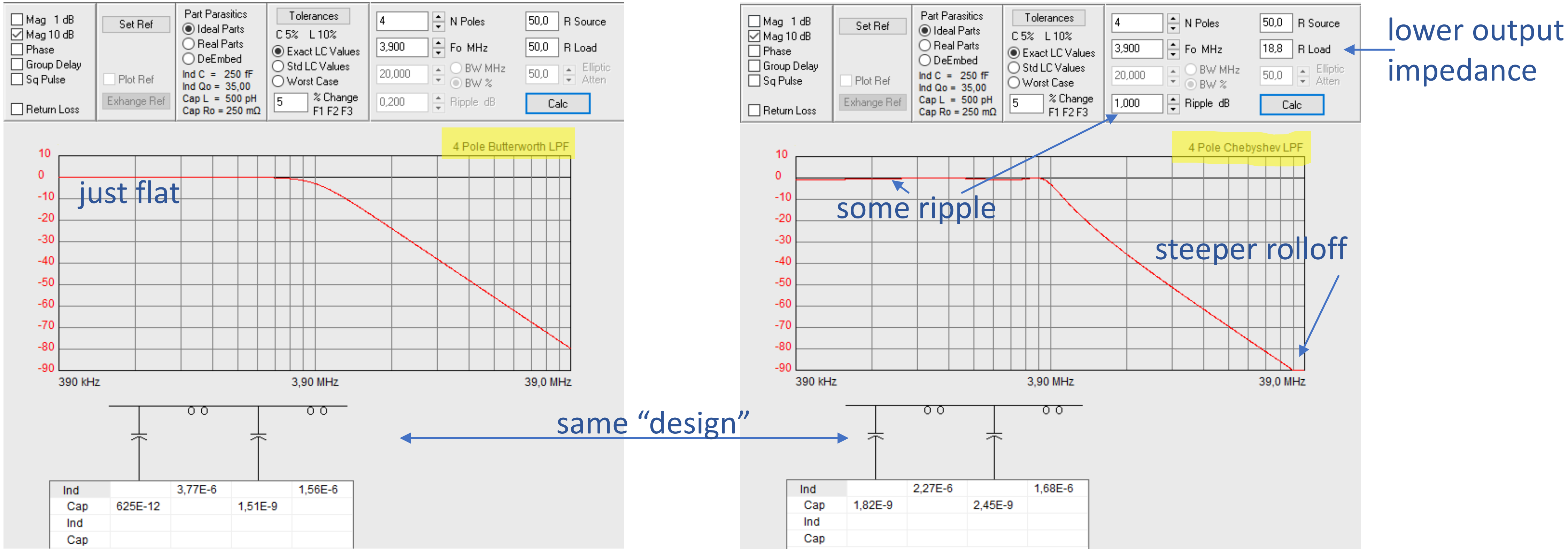
$$f_c = \frac{1}{2\pi RC}$$

https://www.electronics-tutorials.ws/filter/second-order-filters.html

To the bench…

Filter types

From “Iowa Hills RF Filter Designer Version 2.2”, we find for a lowpass, fc = 3.9MHz, 50Ω, the following answers:



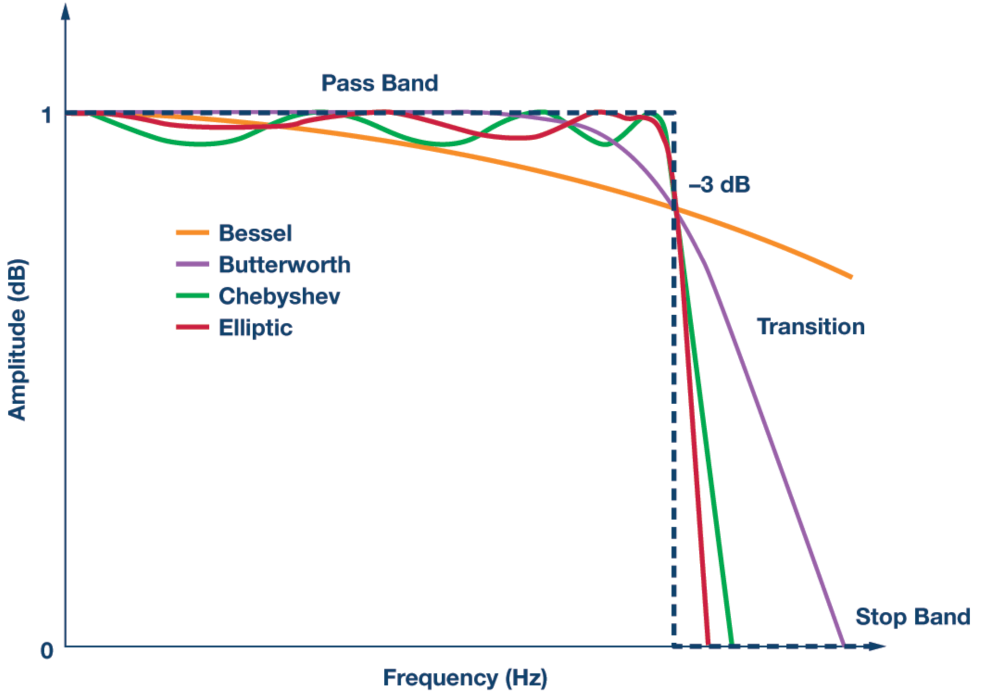
Using different coefficients for the calculation of the filter, forces the same filter “design” to perform electrically different.

used here: http://www.iowahills.com/8DownloadPage.html

nice online tool: https://rf-tools.com/lc-filter/

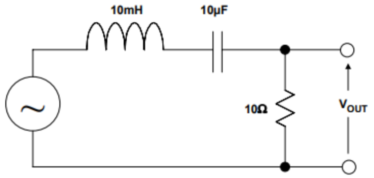
Typical Filter Characteristics

	Advantages	Disadvantages
Butterworth	flat in passband	ringing in step response
Chebyshev	better attenuation pass band	ripple in passband ringing in step response
Bessel	excellent step response	passband not so flat



$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

s = jω

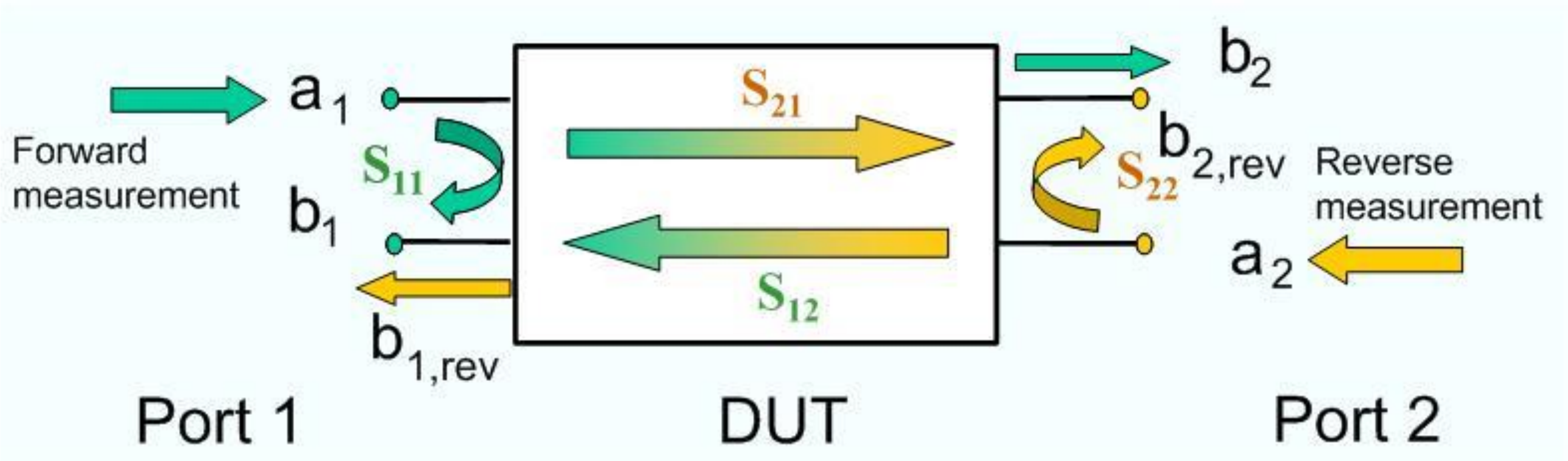


$$\rightarrow \quad H(s) = \frac{Vo}{Vin} = \frac{RCs}{LCs^2 + RCs + 1}$$

Damien Prêle. Advanced Electronic Systems . Master. Advanced Electronic Systems, Hanoi, Vietnam. 2016, pp.140. ffcel-00843641v5f
https://www.allaboutcircuits.com/technical-articles/an-introduction-to-filters/

Measuring filters: S-parameters of a 4-poles

S-parameters are the basic measured quantities of a network analyzer. They describe how the DUT modifies a signal that is transmitted or reflected in forward or reverse direction. For a 2-port measurement the signal flow is as follows.



S11 is the input port voltage reflection coefficient
S12 is the reverse voltage gain
S21 is the forward voltage gain
S22 is the output port voltage reflection coefficient

Measuring filters: S-parameters and VSWR

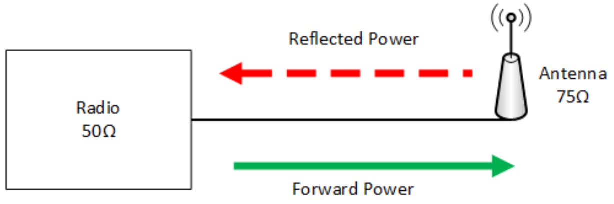
Return Loss can be thought of as a measure of how close the actual input/output impedance of the network is to the nominal system impedance value.

$$RL_{\text{in}} = 10\log_{10}\left|\frac{1}{S_{11}^2}\right| = -20\log_{10}|S_{11}|\ \text{dB} \qquad RL_{\text{out}} = -20\log_{10}|S_{22}|\ \text{dB}.$$

VSWR is defined as $\textbf{VSWR} = \dfrac{1+|S_{11}|}{1-|S_{11}|}$

$$\Gamma = \frac{ZL - Z0}{ZL + Z0}$$

VSWR – Mismatched Impedance



Satoms.com

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$VSWR = \frac{Vmax}{Vmin}$$

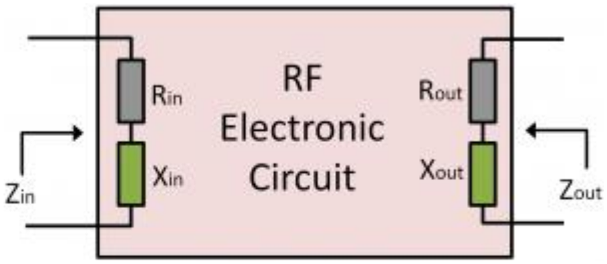
satoms.com

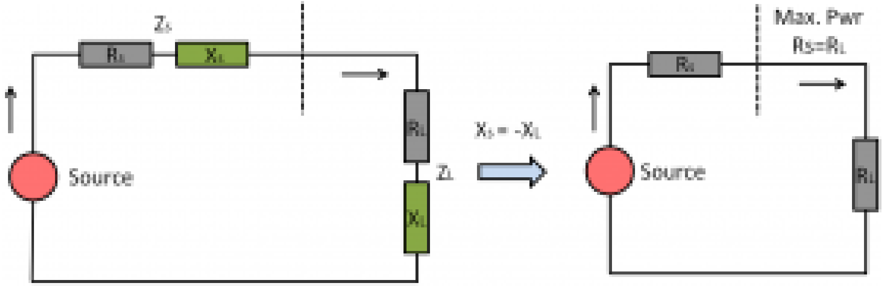
VSWR and Reflected Power (%)

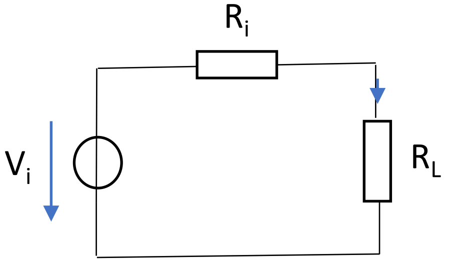
VSWR	% of Reflected Power
1.0	0
1.5	4
2.0	11.1
3.0	25
4.0	36
5.0	44
6.0	51
7.0	56.3
8.0	60.5
9.0	64
10	66.9
15	76.6
20	81.9
	satoms.com

Measuring filters: S-parameters and VSWR

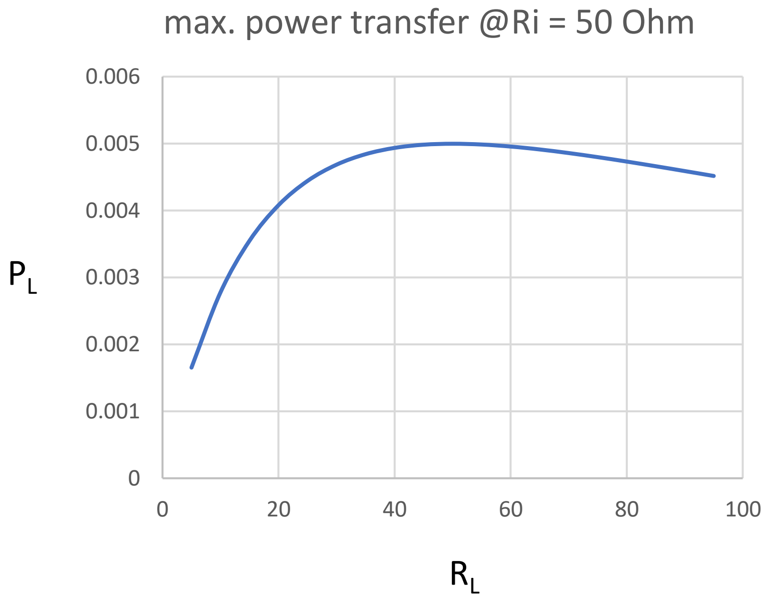
your filter







$$P_L = \left(\frac{V_i}{R_i + RL}\right)^2 * R_L$$



https://handsonrf.com/rf-circuits-impedance-matching-basics/

Nano VNA

Frequency: 50kHz-3GHz
RF output power: -9dBm (error within 1GHz: ±1dB)
Frequency accuracy: <0.5ppm
S21 dynamic ranges: 70dB (1.5GHz), 60dB (3GHz)
S11 dynamic ranges: 50dB (1.5GHz), 40dB (3GHz)
2-port return loss: 20dB typ (1.5GHz), 13dB min (3GHz)
Number of calibration points: 101 points
Number of scanning points: 101 points
Number of traces: 4
Number of mark points: 4 (the mark point bar can be moved up and down on the screen)
USB interface: USB Type-C communication mode: CDC (serial)

Analog Devices: https://www.youtube.com/watch?v=j0K4yboux7g